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INSTABILITY OF ELASTOPLASTIC PLANE FLOWS

G. A. Korolev

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Under conditions of high-speed elastoplastic deformation material flow inhomogeneities associated both with the presence of elastic forces, which may cause self-oscillating processes, and with localized adiabatic heating on a narrow interval of the highest strain rates, are found to occur [1]. Thermoplastic shear was investigated mathematically in [2, 3], and a model of an elastoviscous fluid was considered in [4]. Here, the case of plane elastoplastic flow is studied with allowance for the thermal effects associated with adiabatic conditions and the convective removal of heat from the zone of intense deformation processes.

1. The equation of motion of the medium and the energy balance equation take the form [3, 4]:

$$\frac{\partial V}{\partial T} + V_C \frac{\partial V}{\partial Y} = \frac{1}{\rho} \frac{\partial S}{\partial Y}; \tag{1.1}$$

$$\rho c_V \left(\frac{\partial \Theta}{\partial T} + V_C \frac{\partial \Theta}{\partial Y}\right) = \lambda \frac{\partial^2 \Theta}{\partial Y^2} + \beta S \frac{\partial \Gamma}{\partial T}, \tag{1.2}$$

where V, S, Θ, and Γ are the velocity, stress, temperature, and degree of deformation, respectively, Y is a coordinate, T is time, V_C is the convective velocity component, ρ, c_V, and λ are the density, specific heat, and thermal conductivity coefficient, and β = 0.9-0.95 is a coefficient.

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The coupling equation

$$\frac{1}{G}\frac{\partial S}{\partial T}=\frac{\partial V}{\partial Y}-\frac{\partial \Gamma}{\partial T} \tag{1.3}$$

has been constructed with allowance for the elastic component (G is the shear modulus).

The system (1.1)-(1.3) is closed by the rheological equation relating the stress with the temperature, the degree of deformation and the strain rate: $S = f(\Theta, \Gamma, \partial\Gamma/\partial T)$. In particular, we can take [2]

$$S=c\exp\left[-a\left(\Theta-\Theta_0^*\right)\right](b\partial\Gamma/\partial T)^m\,\Gamma^n. \tag{1.4}$$

Here, c, a, b, m, and n are rheological constants (b = 1 sec); Θ_0^* is the initial temperature. We impose on the solutions of the system (1.1)-(1.4) the boundary conditions

$$\begin{gathered} V(T,0)=0.\ V(T,h)=V_0,\\ \lambda(\partial\Theta/\partial Y)_{Y=0}=\alpha_0\left[\Theta(T,0)-\Theta_0^*\right],\quad (\partial\Theta/\partial Y)_{Y=h}=0 \end{gathered} \tag{1.5}$$

(V_0 is the constant velocity, α_0 is the heat transfer coefficient, and h is the dimension of the deformation zone).

We write the system of equations (1.1)-(1.5) in the dimensionless form:

$$\partial v/\partial t+P\partial v/\partial y=A\partial\sigma/\partial y; \tag{1.6}$$

$$\partial\theta/\partial t+P\partial\theta/\partial y=(1/\mathrm{Bi})\partial^2\theta/\partial y^2+\varkappa\sigma\partial\gamma/\partial t; \tag{1.7}$$

$$\partial\sigma/\partial t=\partial v/\partial y-\partial\gamma/\partial t; \tag{1.8}$$

$$\sigma=\delta\exp(-\theta)(\partial\gamma/\partial t)^m\gamma^n; \tag{1.9}$$

$$\begin{gathered} v(t,0)=0,\ v(t,1)=1,\\ (\partial\theta/\partial y)_{y=0}=\mathrm{Bi}\,\theta(t,0),\ (\partial\theta/\partial y)_{y=1}=0. \end{gathered} \tag{1.10}$$

The dimensionless variables take the values: $t = T/T_0$, $y = Y/h$, $\gamma = \Gamma/\Gamma_1$, $v = V/V_0$, $\sigma = S/(GDT_0)$, $\theta = a(\Theta - \Theta_0^*)$, $T_0 = c_V\rho h/\alpha_0$, $\Gamma_1 = T_0 D$, $D = V_0/h$, $A = c_V{}^2\rho G/\alpha_0{}^2$, $\kappa = \beta c_V\rho a V_0{}^2 G/\alpha_0{}^2$, $\delta = c\alpha_0(bD)^m\Gamma_1{}^n/(G \times V_0 c_V\rho)$, $\mathrm{Bi} = \alpha_0 h/\lambda$, $P = V_C T_0/h = \mathrm{Pe}/\mathrm{Bi}$ (Pe and Bi are the Péclet and Biot numbers).

2. When P = 0 the system (1.6)-(1.8) can be simplified:

$$A\frac{\partial^2\sigma}{\partial y^2}=\frac{\partial^2\sigma}{\partial t^2}+\frac{\partial^2\gamma}{\partial t^2}; \tag{2.1}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\mathrm{Bi}}\frac{\partial^2\theta}{\partial y^2}+\varkappa\sigma\frac{\partial\gamma}{\partial t}. \tag{2.2}$$

Superimposing on the basic solution $\mathbf{U}_0$ small perturbations of the form:

$$\mathbf{U}=\mathbf{U}_0+\mathbf{U}'=\{\sigma_0,\theta_0,\gamma_0\}+\{\sigma',\theta',\gamma'\}=\{\sigma_0,\theta_0,\gamma_0\}+\{\sigma^*,\theta^*,\gamma^*\}\exp(\alpha t+iky)$$

and linearizing the system (2.1), (2.2) with respect to the perturbations, we have

$$A\frac{\partial^2\sigma'}{\partial y^2}=\frac{\partial^2\sigma'}{\partial t^2}+\frac{\partial^2\gamma'}{\partial t^2}; \tag{2.3}$$

$$\frac{\partial\theta'}{\partial t}=\frac{1}{\mathrm{Bi}}\frac{\partial^2\theta'}{\partial y^2}+\varkappa\sigma_0\frac{\partial\gamma'}{\partial t}+\varkappa\sigma'\frac{\partial\gamma_0}{\partial t}. \tag{2.4}$$

Starting from the existence of a nontrivial solution for γ^*, θ^*, having first eliminated the stress perturbation amplitude

$$\sigma^*=Q_0\gamma^*+\alpha R_0\gamma^*-P_0\theta^*$$

$$(Q_0=(\partial\sigma/\partial\gamma)_0=n\sigma_0/\gamma_0,\ R_0=(\partial\sigma/\partial\dot{\gamma})_0=m\sigma_0/\dot{\gamma}_0,$$

$$P_0 = -(\partial\sigma/\partial\theta)_0 = \sigma_0,\ \dot{\gamma}_0 \approx \partial\gamma_0/\partial t)$$

from the system (2.3), (2.4), we obtain the characteristic equation [3]

$$\begin{gathered}\alpha^4 R_0 + \alpha^3(1 + Q_0 - P_0\varkappa\sigma_0 + k^2R_0/\mathrm{Bi}) + \\ + \alpha^2[\varkappa\dot{\gamma}_0 P_0 + k^2(1/\mathrm{Bi} + Q_0/\mathrm{Bi} + AR_0)] + \\ + \alpha k^2(Ak^2R_0/\mathrm{Bi} + AQ_0 - P_0A\varkappa\sigma_0) + k^4AQ_0/\mathrm{Bi} = 0.\end{gathered} \tag{2.5}$$

From the Routh–Hurwitz conditions it follows that as $k \to \infty$ the roots of the Eq. (2.5) α will always be negative, i.e., the system is stable. For $k = 0$ when the inequality

$$P_0\varkappa\sigma_0/(1 + Q_0) > 1 \tag{2.6}$$

is satisfied the system loses stability.

For determining the maximum of α we equate the derivative $d\alpha/dk^2$ to zero and find

$$k^2 = \frac{(P_0A\sigma_0\varkappa/R_0 - AQ_0/R_0) - \alpha\,[(1 + Q_0)/(\mathrm{Bi}\,R_0) + A] - \alpha^2\,\mathrm{Bi}}{2\,(\alpha A/\mathrm{Bi} + A/\mathrm{Bi})}. \tag{2.7}$$

Having investigated the characteristic equation (2.5) for the extremum, taking into account the fact that the expression (2.7) is always positive, we have one more criterion of instability:

$$P_0\sigma_0\varkappa > Q_0 + 2\sqrt{\varkappa\dot{\gamma}_0P_0Q_0/(A\,\mathrm{Bi})}. \tag{2.8}$$

The satisfaction of inequality (2.8) indicates the predominance of softening of the material over strain hardening; in this case we have the estimate for the maximum of the eigenvalue α:

$$\alpha \leqslant M = \frac{2M_1}{\sqrt{(M_2 + R_0)^2 + 4R_0M_1/(A\,\mathrm{Bi})} + (M_2 + R_0)}$$
$$(M_1 = P_0\sigma_0\varkappa - Q_0,\ M_2 = (1 + Q_0)/(A\mathrm{Bi})).$$

Since in most cases of practical importance the quantity $\varepsilon = 1/(A\ \mathrm{Bi}) \ll 1$, M can be expanded in powers of the small quantity ε:

$$M = \frac{P_0\sigma_0\varkappa - Q_0}{R_0}\left[1 + \frac{\varepsilon}{R_0}(P_0\sigma_0\varkappa + 1)\right] + O(\varepsilon^2).$$

We write (2.6) and (2.8) in dimensional form:

$$\frac{\beta a S_0}{\rho c_V} > G + \frac{n}{\dot{\gamma}_0}\sigma_0; \tag{2.9}$$

$$\frac{\Gamma_0 a\beta S}{\rho c_V n} > 1 + 2\sqrt{\frac{\beta\dot{\Gamma}_0\lambda\Gamma a}{\rho c_V^2 n}}. \tag{2.10}$$

The inequality (2.10) can be simplified by taking into account the smallness of the second term on the right and the relation (1.4):

$$\Gamma^{n+1} > \frac{nc_V\rho\exp\left[a\left(\Theta - \Theta_0^*\right)\right]}{a\beta c\dot{\Gamma}_0^m}.$$

By strengthening the latter inequality it is easy to obtain the criterion proposed in [2].

Inequalities (2.9) and (2.10) determine the intervals of values of the parameters of the deformation processes for which loss of stability is possible. The former determines the instability associated with the developed elastic properties of the material, which is expressed as periodic falls in stress with simultaneous increase in temperature. The occurrence of self-oscillation depends to a large extent on the quantity G. The domain of reali-

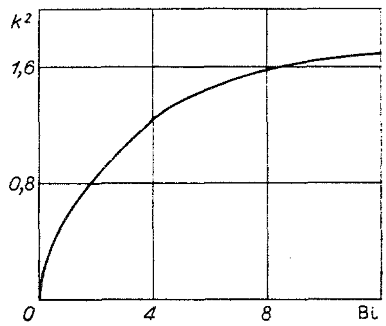

Fig. 1

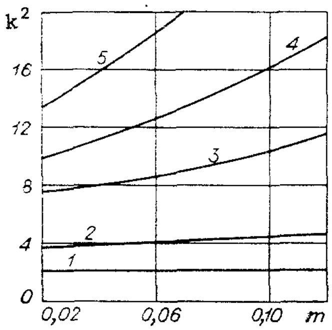

Fig. 2

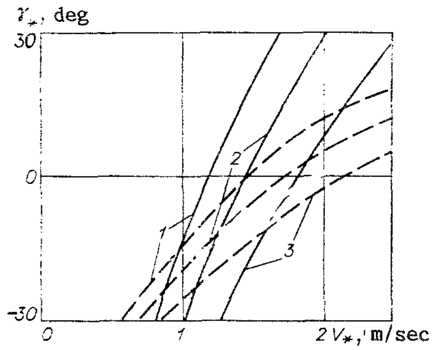

Fig. 3

zation of the criterion (2.9) is contained in the domain of the parameters satisfying the criterion (2.10), which is associated with the possibility of the stability of the flow being disturbed as a result of plastic heating after a certain degree of deformation has been reached.

3. As follows from (2.10), for P = 0, n = 0 the necessary condition of loss of stability is satisfied, i.e., strain saturation leads to a flow state in which a further increase in the degree of deformation will not lead to a subsequent hardening of the material. The general (sufficient) instability criterion will contain the wave number k, which depends on the boundary conditions to which the basic solution (1.10) is subjected. From the characteristic equation (2.5) and (1.9) there follows

$$(\dot{\gamma}_0)^{m+1} > \frac{k^2 m \exp\theta_0}{\mathrm{Bi}\,\varkappa\delta}. \qquad (3.1)$$

The sufficient criterion for self-oscillation takes the form:

$$\varkappa\delta^2 > \frac{k^2 m\delta \exp\theta_0}{\mathrm{Bi}\,(\dot{\gamma}_0)^{m+1}} + \frac{\exp 2\theta_0}{(\dot{\gamma}_0)^{2m}}. \qquad (3.2)$$

From (3.1), (3.2) we obtain the averaged estimates: $\kappa\delta > k^2m/\mathrm{Bi}$, $\kappa\delta > k^2m/\mathrm{Bi} + 1/\delta$. Using the averaging $\dot{\Gamma} \sim V_0/h$, we can strengthen the inequality (3.1) and estimate the critical velocity sufficient for the development of instability:

$$V^* > \frac{h}{b}\left(\frac{k^2 m\lambda b}{h^2 ac\beta}\right)^{1/(1+m)}. \qquad (3.3)$$

In the case of metal deformation the hardening parameter m is fairly small (m < 0.1) and the estimate (3.3) can be simplified:

$$V^* > (k^2 m\lambda)/(ach\beta). \qquad (3.4)$$

For testing the stability of the steady-state solutions, found analytically from the basic system of equations (as $t \to \infty$), we used the Galerkin method. In order to improve the accuracy of the calculations, we chose an orthogonal system of basis functions for the series expansion of the small perturbations superimposed on the solutions. The eigenvalues of the differential operator matrix were found after reducing it to almost diagonal form. The dependence of k^2 on Bi is shown in Fig. 1.

4. The steady-state solution of the system (1.6)-(1.10) with n = 0 and constant P is found as a result of the numerical integration of the initial system of equations by the matrix sweep iteration method. In practice, the values of P are small (often $P \ll 1$), which makes it possible to carry out the expansion in a small parameter in the same way as in [5] and thus simplify the numerical realization of the problem by reduction to the usual Runge–Kutta scheme for determining the second term of the expansion. As follows from the numerical calculations using the Galerkin method, the convective removal of heat from the deformation zone leads to a considerable narrowing of the parametric interval of existence of instability. Using (3.4) as the rheological relation, we calculated k^2 as a function of m and the Péclet number. The results of the calculations are reproduced in Fig. 2: the curves 1-5 correspond to Pe = 0.1, 2, 5, 7, and 10. On the basis of the above-mentioned calculations we determined the critical speeds of metal working by continuous orthogonal cutting necessary for the formation of a sawtoothed (cyclic) [6] chip. Figure 3 shows the limits of transition to unstable chip formation associated with the localized thermoplastic shear effect [1] for 2Kh18N9T steel (when m = 0.06) as a function of the values of the working speed V_*, the principal leading cutter angle γ_* and φ, the angle of inclination of the conventional shear plane. Curves 1-3 correspond to $h = 1.2\cdot10^{-4}$, $1.0\cdot10^{-4}$, and $0.8\cdot10^{-4}$ m – the linear dimension of the interval of principal strains in the chip formation zone (equal to approximately one tenth of the thickness of the cut); $\varphi = 20$ and 35° for the continuous and broken curves, respectively. The regions of unstable cutting lie to the right of the curves.

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